

[mex200] Action-angle coordinates of plane pendulum: librations

Determine the canonical transformation $(\phi, p) \rightarrow (\theta, J)$ which produces the action-angle coordinates for the librational motion of the plane pendulum:

$$H(\phi, p) = \frac{p^2}{2m} + G(1 - \cos \phi), \quad M \doteq m\ell^2, \quad G \doteq mg\ell.$$

(a) Find the action $J(E)$, the angular frequency $\omega(E)$, and the angle coordinate $\theta(\phi, J)$. (b) Use this result to determine $\phi(t)$ in closed form.

Solution: