[mex202] Hamilton-Jacobi theory for projectile motion

Consider a particle of mass m moving in a uniform vertical gravitational field:

$$H = \frac{1}{2m}(p_x^2 + p_y^2) + mgy.$$

(a) Find Hamilton's principal function $S(x, y, P_1, P_2, t)$ as the solution of the Hamilton-Jacobi equation. (b) Derive the solutions $x(t), y(t)$ from $S(x, y, P_1, P_2, t)$ for initial conditions $x(0) = y(0) = 0$, $\dot{x}(0) = \dot{x}_0, \dot{y}(0) = \dot{y}_0$.

Solution: