

## [mex212] Optical birthday cards

Jack and Jill are twins. They synchronize their watches (and calendars) to  $t = t' = 0$  at noon on their 20th birthday. Then Jill travels into space at  $v = 0.8c$ , turns around, and returns at the same speed. When they are reunited Jack has aged 10 years and Jill 6 years as analyzed in [mln56].

While they are separated, each twin sends a light signal at noon on their birthday according to the local calendar. When Jack is about to send his 10th card and Jill her 6th, they find themselves reunited.

- (a) Use the Doppler effect to determine the dates (measured in units of local years) when each birthday card arrives.
- (b) Use the Minkowski diagram of [mln56] to draw the world lines traced by each birthday card in confirmation of the result obtained via the Doppler effect.

**Solution:**