Jack and Jill are twins. They synchronize their watches (and calendars) to \( t = t' = 0 \) at noon on their 20th birthday. Then Jill travels into space at \( v = 0.8c \), turns around, and returns at the same speed. When they are reunited Jack has aged 10 years and Jill 6 years as analyzed in [mln56]. While they are separated, each twin sends a light signal at noon on their birthday according to the local calendar. When Jack is about to send his 10th card and Jill her 6th, they find themselves reunited.

(a) Use the Doppler effect to determine the dates (measured in units of local years) when each birthday card arrives.

(b) Use the Minkowski diagram of [mln56] to draw the world lines traced by each birthday card in confirmation of the result obtained via the Doppler effect.

Solution: