[mex229] Growth of falling raindrop

A spherical raindrop of mass density $\rho_W$ falling through fog of mass density $\rho_F$ accumulates mass by absorbing all fog droplets (assumed stationary) in its way. The initial radius is $r_0$ and the initial velocity is zero. The acceleration due to gravity is $g$. Air resistance is to be neglected.

(a) Relate the radial growth $\dot{r}$ of the raindrop to its instantaneous velocity $v$.
(b) Construct a differential equation (nonlinear second order ODE) for the radius $r$ of the raindrop.
(c) Show that the acceleration $\dot{v}$ is initially equal to $g$ and approaches the asymptotic value $g/7$.
(d) Plot $\dot{v}$ versus $t$ for $0 \leq t \leq 2.5$ and $\rho_W/\rho_F = 1000$, $g = 10$, $r_0 = 0.001$ (all in SI units).

Solution: