Pendulum under forced rotation

Consider a pendulum consisting of two masses $m_1$ and one mass $m_2$ connected by four rods of negligible mass and length $L$. Mass $m_2$ is constrained to move along the vertical axis. The masses $m_1$ are forced to rotate with constant angular velocity $\Omega$ about the vertical axis.

(a) Determine the Lagrangian $L(\theta, \dot{\theta})$ and derive the Lagrange equation for the variable $\theta$.

(b) Determine the condition for the existence of a stable rotating mode with nonzero $\theta = \theta_0 = const$ and determine the dependence of $\theta_0$ on $\Omega$.

Solution: