Cometary motion on hyperbolic orbit

Determine a parametric representation $x(\xi), y(\xi), t(\xi), \vartheta(\xi)$ for the hyperbolic motion in time of a comet with mass $m$ in the central force potential $V(r) = -\kappa/r$. Start from the general integral expression for $t(r)$ and use the parametrization $\tilde{a} + r = \tilde{a}e \cosh \xi$ with $\tilde{a} = \kappa/2E$ and $e^2 = 1 + 2E\ell^2/m\kappa^2$, where $E$ is the energy and $\ell$ the angular momentum.

Solution: