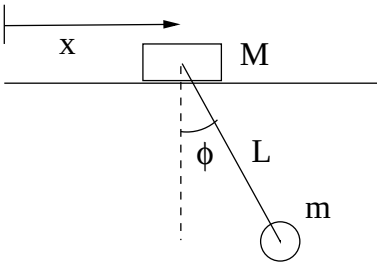
## [mex24] Pendulum with sliding pivot: Lagrange equations

A block of mass $M$ is free to slide horizontally on an airtrack with negligible friction. Suspended from the block by a rod of negligible mass and length $L$ is a mass $m$ swinging in a vertical plane.
(a) Determine the Lagrangian $L(x, \phi, \dot{x}, \dot{\phi})$ and derive the Lagrange equations for $x$ and $\phi$.
(b) Determine the angular frequency $\omega_0$ of small-amplitude oscillations of this system.




**Solution:**