Elastic collision: velocities of scattered particles

A particle of mass $m_1$ and incident velocity $\vec{v}_0$ undergoes an elastic collision via central force with a particle of mass $m_2$ that is initially at rest. Show that the velocities of the scattered particles depend on the scattering angles in the laboratory frame as follows:

$$\frac{\vec{v}_2}{\vec{v}_0} = 2 \frac{m}{m_2} \cos \theta_2, \quad \frac{\vec{v}_1}{\vec{v}_0} = \frac{m}{m_2} \left( \cos \theta_1 \pm \sqrt{\frac{m_2^2}{m_1^2} - \sin^2 \theta_1} \right),$$

where two solutions ($\pm$) exist for $m_1 > m_2$ and one solution ($+$) for $m_1 < m_2$.

Solution: