

[mex245] Classical inverse scattering problem III

The final step in the reconstruction of the (central force) scattering potential $V(r)$ from the observed scattering cross section $\sigma(\theta)$ as outlined in [mln104] involves the integration of the differential relation

$$\frac{\pi}{w}dw = -d\left(\frac{u}{w}\right) \int_0^{u^2/w^2} dx \frac{\tilde{\theta}'(x)}{\sqrt{u^2/w^2 - x}}$$

into the integral equation

$$\Rightarrow w(u) = \exp\left(\frac{1}{\pi} \int_{w/u}^{\infty} ds \frac{\theta(s)}{\sqrt{s^2 - [w(u)]^2/u^2}}\right)$$

for the quantity $w(u) \doteq \sqrt{1 - V(1/u)/E}$ with $u = 1/r$. Carry out this step by interchanging the order of integration on the right.

Solution: