Consider a mathematical pendulum (mass $m$, length $\ell$) with the pivot $P$ oscillating horizontally, $x_P = A \cos \omega t$. Show that the Lagrangian is

$$L = \frac{1}{2} m \ell^2 \dot{\phi}^2 + m A \omega^2 \ell \cos \omega t \sin \phi + m g \ell \cos \phi.$$