Consider a mathematical pendulum (mass $m$, length $\ell$) with the pivot $P$ oscillating vertically, $y_P = A \cos \omega t$. Show that the Lagrangian is

$$L = \frac{1}{2} m \ell^2 \dot{\phi}^2 + mA \omega^2 \ell \cos \omega t \cos \phi + m g \ell \cos \phi.$$