[mex261] Harmonic oscillator with attenuation

The equation of motion of a harmonic oscillator with attenuation is written in the form

$$\ddot{x} + \alpha_m \,\mathrm{sgn}(\dot{x})|\dot{x}|^m + \omega_0^2 x = 0,$$

where $\omega_0^2 = k/m$ is the angular frequency of the undamped oscillator and $\mathrm{sgn}(\dot{x})$ denotes the sign ($\pm$) of the instantaneous velocity. Here we consider the three cases of Coulomb damping ($m = 0$), linear damping ($m = 1$), and quadratic damping ($m = 2$). Use $\omega_0 = 1$ throughout.
Employ the Mathematica options of NDSolve and ParametricPlot to numerically solve the equation of motion for all three cases and to plot x versus $\dot{x}$ for initial conditions $x(0) = 9$ and $\dot{x}(0) = 0$. Vary the attenuation parameter α_m in each case and watch out for qualitative changes in the phase-plane trajectory. Present a collection of neat graphs that emphasize the differences between the three cases as well as the differences between parameter regimes for one or the other case. Describe the different types of trajectories.

Solution: