[mex263] Driven harmonic oscillator with Coulomb damping

The harmonic oscillator with Coulomb damping and harmonic driving force is described by the equation of motion,

$$\ddot{x} + \alpha\,\mathrm{sgn}(\dot{x}) + \omega_0^2 x = A\cos(\omega t), \tag{1}$$

where $\omega_0^2 = k/m$, $\alpha = \mu/m$, $A = F_0/m$. The function $\mathrm{sgn}(\dot{x})$ denotes the sign ($\pm$) of the instantaneous velocity. The oscillator has mass m and the spring has stiffness k. The coefficient of kinetic (and static) friction is μ. The natural angular frequency of oscillation is ω_0. The harmonic driving force has amplitude F_0 and angular frequency ω. In this project we consider an oscillator at resonance ($\omega = \omega_0 = 1$) launched from $x(0) = 0$ with initial velocity $\dot{x}(0) = v_0 > 0$.

(a) Use the DSolve option of Mathematica to determine the analytic solutions of (1) with the given initial conditions, valid over a time interval with $\dot{x} > 0$. Check whether the solution that Mathematica gives you can be further simplified by hand. Use the ParametricPlot option of Mathematica to plot this solution in the phase plane, i.e. x versus $\dot{x}$. Use $A = 1$, $v_0 = 9$ and various values of α (all in SI units).

(b) Use the NDSolve and ParametricPlot options of Mathematica to generate and plot data for the solution of (1) over a larger time interval. Use again $A = 1$, $v_0 = 9$ and various values of α. Tune α to a value that yields a periodic trajectory.

(c) Investigate the stability of the periodic trajectory thus found numerically. Is it a limit cycle? Vary the initial conditions and check whether the periodic trajectory attracts or repels nearby trajectories.

Solution: