Consider the Lagrangian $L(z, \dot{z}) = \frac{1}{2}m\dot{z}^2 - mgz$ of a particle with mass $m$ moving vertically in 3D space under the influence of a uniform gravitational field. Show that the transformation $X = x, Y = y, Z = z + \epsilon t$ is a symmetry transformation by establishing the relation

$$L'(Z, \dot{Z}, t, \epsilon) = L(Z, \dot{Z}) + \frac{d}{dt}F(Z, t, \epsilon).$$

Find the function $F(Z, t, \epsilon)$ and the conserved quantity $I(z, \dot{z}, t)$ associated with this symmetry.

Solution: