In search of some hyperbolic orbit

A particle of unit mass \( m = 1 \) moves from infinity along a straight line which, if continued, would allow it to pass a distance \( d = b\sqrt{2} \) from a point \( P \). Instead, the particle is attracted toward \( P \) by the central force \( F(r) = -\frac{k}{r^5} \). If the angular momentum of the particle relative to \( P \) is \( \ell = \sqrt{k}/b \), show that the orbit is \( r(\theta) = b \coth(\theta/\sqrt{2}) \).

Solution: