Kepler’s second and third laws

Derive Kepler’s second and third laws of planetary motion from the results established in class for central force motion. Use the case of an elliptic orbit (0 < e < 1). Specifically: (a) Show that the rate at which area is swept by the position vector of the planet, \( dA/dt \), is a constant. Determine that constant. (b) Integrate the result for \( dA/dt \) over one period of revolution \( \tau \) to show that the following relation holds between \( \tau \) and the semi-major axis \( a \): \( \tau^2 = 4\pi^2 (m/\kappa) a^3 \), where \( \kappa = GMm, M = m_S + m_P, m = m_S m_P / (m_S + m_P) \).

Solution: