Consider the central force potential $V(r) = -\kappa/r^2$. If $\kappa > \ell^2/2m$, all orbits at $E > 0$ are unbounded and all orbits at $E < 0$ are bounded. (a) Show that these orbits can be expressed in the form

$$E > 0 : \frac{1}{r} = \sqrt{\frac{2mE}{2m\kappa - \ell^2}} \sinh \left( \vartheta \sqrt{\frac{2m\kappa}{\ell^2} - 1} \right), \quad E < 0 : \frac{1}{r} = \sqrt{\frac{2m|E|}{2m\kappa - \ell^2}} \cosh \left( \vartheta \sqrt{\frac{2m\kappa}{\ell^2} - 1} \right).$$

(b) Determine the time it takes the particle to move along the bounded orbit from $r_{\text{max}}$ to the center of force ($r = 0$).

Solution: