[mex48] Orbital differential equation applied to Kepler problem

Derive the orbital relation $p/r = 1 + e\cos(\vartheta - \vartheta_0)$ with $p = \ell^2/m\kappa$ and $e = \sqrt{1 + 2E\ell^2/m\kappa^2}$, which describes all orbits of the Kepler problem, from the orbital differential equation

$$\frac{d^2u}{d\vartheta^2} + u = -\frac{m}{\ell^2 u^2} F(u^{-1}),$$

where $u \equiv 1/r$, $F(r) = -dV/dr$, and $V(r) = -\kappa/r$. Do not reason backward. Pretend you do not know the solution.

Solution: