Exponential spiral orbit

A particle of mass $m$ moves along an exponential spiral orbit $r(\theta) = r_0 e^\theta$ under the influence of a central force potential $V(r)$. (a) Use the orbital differential equation

$$\frac{d^2 u}{d\theta^2} + u = -\frac{m}{\ell^2 u^2} F(u^{-1}),$$

where $u \equiv 1/r$, $F(r) = -dV/dr$ to determine the potential $V(r)$. (b) Determine the energy $E$ of this orbit. (c) Determine the motion in time $r(t), \theta(t)$ of the particle on this orbit.

Solution: