

[mex49] Exponential spiral orbit

A particle of mass m moves along an exponential spiral orbit $r(\vartheta) = r_0 e^{\vartheta}$ under the influence of a central force potential $V(r)$. (a) Use the orbital differential equation

$$\frac{d^2 u}{d\vartheta^2} + u = -\frac{m}{\ell^2 u^2} F(u^{-1}),$$

where $u \equiv 1/r$, $F(r) = -dV/dr$ to determine the potential $V(r)$. (b) Determine the energy E of this orbit. (c) Determine the motion in time $r(t), \vartheta(t)$ of the particle on this orbit.

Solution: