[mex57] Loss of kinetic energy in elastic collision

Consider a particle of mass $m_1$ and incident velocity $\vec{v}_0$ undergoing an elastic collision via central force with a target of mass $m_2$ that is initially at rest. The particle emerges with velocity $\vec{v}_1$ from the collision as viewed in the laboratory frame. The figure shows this velocity in relation to the center-of-mass velocity $\vec{v}_\text{cm}$ and the final velocity $\vec{v}_1$ of the particle in the center-of-mass frame. Also shown are the scattering angles $\theta$ (center-of-mass frame) and $\bar{\theta}$ (laboratory frame). Show that the ratio of the final and initial kinetic energies in the laboratory frame is

$$\frac{T_1}{T_0} = \frac{1 + 2\rho \cos \theta + \rho^2}{(1 + \rho)^2}, \quad \rho = \frac{m_1}{m_2}.$$ 

Solution: