[mex64] Foucault pendulum

Consider a location at northern latitude λ on the Earth's surface. A pendulum (mass m, length L) is free to swing in any direction. At time $t = 0$, the pendulum is set in motion from a small displacement $x_0 > 0$, $y_0 = 0$ with no initial velocity. (a) Show that the linearized equations of motion including the effect of the Coriolis force can be expressed in the form

$$\ddot{q} + 2i\omega_z \dot{q} + \Omega^2 q = 0; \qquad q \equiv x + iy, \quad \Omega = \sqrt{g/L}, \quad \omega_z = \omega \sin\lambda,$$

where ω is the angular frequency of the Earth's rotation. This equation of motion describes a harmonic oscillator with imaginary damping. (b) Show that for the initial conditions stated above and for $\omega_z \ll \Omega$ its solution is of the form $q(t) = x_0 \cos\Omega t e^{-i\omega_z t}$. (c) Show that the last factor in this solution describes a precession with angular frequency ω_z of the plane in which the pendulum swings.

Solution: