

[mex69] Parallel-axis theorem

Consider a rigid body composed of particles with mass m_α whose relative positions have constant magnitude. The total mass of the rigid body is $\sum_\alpha m_\alpha = M$. Two body coordinate systems with parallel axes are labelled (r_1, r_2, r_3) and (R_1, R_2, R_3) , respectively. The center of mass coordinate of the rigid body is $(0, 0, 0)$ in the former and (a_1, a_2, a_3) in the latter. Show that the inertia tensor of the rigid body in the displaced coordinate system, I_{ij} , is obtained from the inertia tensor in the center-of-mass system, $I_{ij}^{(cm)}$, by adding to it the inertia tensor of a point mass M at the position of the center of mass:

$$I_{ij} = I_{ij}^{(cm)} + M (a^2 \delta_{ij} - a_i a_j).$$

Solution: