

[mex76] Velocity-dependent central force

A particle moves under the influence of a velocity-dependent central force

$$F = \frac{1}{r^2} \left(1 - \frac{\dot{r}^2 - 2r\ddot{r}}{c^2} \right),$$

where c is a constant. (a) Show that the Lagrangian and Hamiltonian of this system can be expressed as follows:

$$L(r, \dot{r}, \dot{\vartheta}) = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\vartheta}^2) - \frac{1}{r} \left(1 + \frac{\dot{r}^2}{c^2} \right), \quad H(r, p, \ell) = \frac{p^2}{2(m - 2/c^2 r)} + \frac{\ell^2}{2mr^2} + \frac{1}{r}.$$

(b) Derive the Lagrange equations from L and the canonical equations from H and show that they are equivalent.

Solution: