A harmonic oscillator (mass $m$, spring constant $k$) is attached to a cart that moves with constant velocity $v_0$. Describe the dynamics in the coordinate system ($x$) that is at rest and in the coordinate system ($x'$) that is moving with the cart.

(a) Construct the Lagrangian $L$ of the oscillator in the rest frame and derive the associated Lagrange equation. Construct the Hamiltonian $H$ from $L$.

(b) Construct the Lagrangian $L'$ of the oscillator in the moving frame and derive the associated Lagrange equation. Construct the Hamiltonian $H'$ from $L'$.

(c) Show that the Lagrange equations obtained in (a) and (b) are equivalent.

(d) Which of the two quantities $H, H'$, if any, represents the total energy of the oscillator?

(e) Which of the two quantities $H, H'$, if any, represents a conserved quantity?

Solution: