[mex80] Effect of point transformation on Hamiltonian

Consider the point transformation $q_i = q_i(Q_1, \ldots, Q_n, t), i = 1, \ldots, n$ between two sets of generalized coordinates. The original and transformed Lagrangians are $L(q_1, \ldots, q_n, \dot{q}_1, \ldots, \dot{q}_n, t) = \tilde{L}(Q_1, \ldots, Q_n, \dot{Q}_1, \ldots, \dot{Q}_n, t)$. By comparing the differentials dL and $d\tilde{L}$ show that the following relations hold between the canonical momenta $\{P_i\}, \{p_i\}$ and between the Hamiltonians $H, \tilde{H}$ before and after the transformation:

$$\begin{aligned} P_i(Q_1, \ldots, Q_n, p_1, \ldots, p_n, t) &= \sum_j p_j \frac{\partial q_j}{\partial Q_i}, \\ \tilde{H}(Q_1, \ldots, Q_n, P_1, \ldots, P_n, t) &= H(q_1, \ldots, q_n, p_1, \ldots, p_n, t) - \sum_j p_j \frac{\partial q_j}{\partial t}. \end{aligned}$$

Solution: