

[mex83] Time-dependence of generating functions

Consider the generating functions $F_1(q, Q, t)$ and $F_2(q, P, t) = F_1 + QP$ of a time-dependent canonical transformation $Q = Q(q, p, t)$, $P = P(q, p, t)$. (a) Show that both generating functions have the same explicit time-dependence: $\partial F_1/\partial t = \partial F_2/\partial t$. (b) Find the generating functions $F_1(q, Q, t)$ and $F_2(q, P, t)$ for the specific transformation, which was shown in [mex84] to be canonical, $Q = \sqrt{p - t^2}$, $P = -2q\sqrt{p - t^2}$, and verify that $\partial F_1/\partial t = \partial F_2/\partial t$ is indeed satisfied.

Solution: