[mex89] Pendulum with string of slowly increasing length

Consider a plane pendulum consisting of a point mass m attached to a string of slowly increasing length $\ell = \ell_0 + \alpha t$. (a) Determine the Lagrangian $L(\phi, \dot{\phi}, t)$ and the Hamiltonian $H(\phi, p, t)$ of this dynamical system. (b) Evaluate the equation of motion for the variable ϕ in the form of a 2^{nd} order ODE from both L and H. Compare this equation of motion with that of a damped pendulum.

Solution: