

[mex92] Action-angle coordinates of an anharmonic oscillator

Determine the canonical transformation $(q, p) \rightarrow (\theta, J)$ which produces the action-angle coordinates for the anharmonic oscillator:

$$H(q, p) = \frac{p^2}{2m} + U \tan^2(\alpha q) \quad \rightarrow \quad K(J).$$

(a) Find the transformed Hamiltonian $K(J)$ and determine the angular frequency $\omega(J)$ which determines the linear time evolution $\theta(t) = \omega(J)t + \theta_0$ of the angle coordinates. (b) Find the transformation relations $q(\theta, J)$, $p(\theta, J)$, which amount to a solution of the dynamical problem.

Solution: