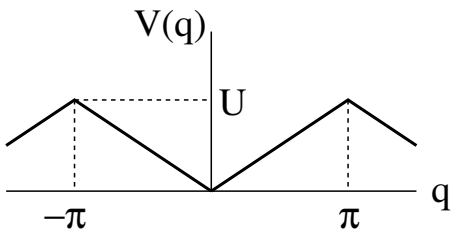
[mex93] Unbounded motion in piecewise linear periodic potential

Consider a particle of mass m moving in a periodic potential $V(q) = (U/\pi)|q|$ for $-\pi \leq q \leq \pi$ and $V(q + 2\pi) = V(q)$. For energies $E > U > 0$, the motion is unbounded and can be reinterpreted as a rotational mode of bounded motion. Solve this dynamical problem via transformation $(q, p) \to (\theta, J)$ to action-angle coordinates by establishing the following relations:

$$p(q,E) = \sqrt{2m\,[E - (U/\pi)|q|]}, \quad J(E) = \frac{2\sqrt{2m}}{3U}\left[E^{3/2} - (E-U)^{3/2}\right],$$
$$\omega(E) = \frac{1}{\sqrt{2m}}\left[\sqrt{E} + \sqrt{E-U}\right], \quad \theta(q,E) = \pm\pi\,\frac{\sqrt{E} - \sqrt{E - (U/\pi)|q|}}{\sqrt{E} - \sqrt{E-U}}, \quad 0 \leq \pm q \leq \pi.$$



Solution: