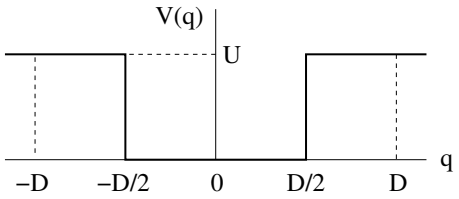
[mex95] Bounded motion in piecewise constant periodic potential

Consider a particle of mass m moving in the potential $V(q) = 0$ for $0 < |q| < D/2$ and $V(q) = U$ for $D/2 < |q| < D$ with periodicity $V(q+2D) = V(q)$. For energies $E < U$ the motion is bounded. Solve this dynamical problem via transformation $(q, p) \to (\theta, J)$ to action-angle coordinates for motion with initial conditions $q(0) = 0,\ p(0) > 0$. (a) Find the function $K(J)$, which expresses the Hamiltonian as a function of the action coordinate. (b) Find the period $T \equiv 2\pi/\omega(J)$ of the librational motion. (c) Find the function $q(\theta, J)$ for $0 < \theta < 2\pi$. (d) Plot in one diagram the functions $J = \text{const}$ and $p(t)$ for $0 < t < T$. (e) Plot in a second diagram the functions $q(t)$ and $\theta(t)$ for $0 < t < T$.



Solution: