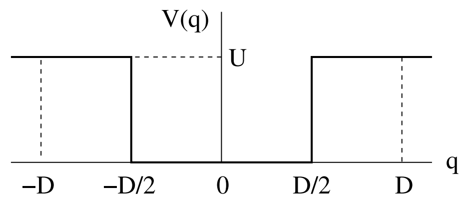
[mex96] Unbounded motion in piecewise constant periodic potential

Consider a particle of mass m moving in the potential $V(q) = 0$ for $0 < |q| < D/2$ and $V(q) = U$ for $D/2 < |q| < D$ with periodicity $V(q + 2D) = V(q)$. For energies $E > U$ the motion is unbounded and can be reinterpreted as a rotational mode of bounded motion. Solve this dynamical problem via transformation $(q, p) \to (\theta, J)$ to action-angle coordinates for motion with initial conditions $q(0) = 0,\ p(0) > 0$: (a) Find the function $J(E)$, which expresses the action as a function of the energy. (b) Find the period $T \equiv 2\pi/\omega(E)$ of the rotational motion. (c) Find the function $\theta(q, E)$ for $0 < q < 2D$. (c) Plot in one diagram the functions $J = \text{const}$ and $p(t)$ for $0 < t < T$. (d) Plot in a second diagram the functions $q(t)$ and $\theta(t)$ for $0 < t < T$.



Solution: