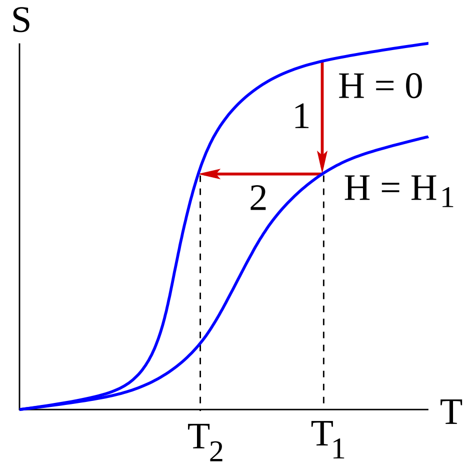
Adiabatic demagnetization [tln24]



Equation of state for paramagnetic salt in a weak magnetic field: $M(T,H) = \chi_T(T)H$.

Helmholtz free energy:

$$dA = -SdT + HdM, \quad \left(\frac{\partial A}{\partial M}\right)_T = H = \frac{M}{\chi_T}$$

$$\Rightarrow \; A(T,M) = A(T,0) + \frac{M^2}{2\chi_T}.$$

Entropy: $S(T,M) = -\left(\frac{\partial A}{\partial T}\right)_M = S(T,0) - \frac{1}{2}M^2\left[\frac{d}{dT}\chi_T^{-1}\right]$

$$\Rightarrow \; S(T,H) = S(T,0) + \frac{1}{2}H^2\frac{d\chi_T}{dT}; \quad \chi_T > 0, \; \frac{d\chi_T}{dT} < 0 \;\text{ for paramagnet.}$$

Third law: $\lim_{T\to 0} S(T,H) = 0$ independent of $H \;\Rightarrow\; \lim_{T\to 0}\frac{d\chi_T}{dT} = 0.$

1. Isothermal magnetization: $\Delta S = \frac{1}{2}\frac{d\chi_T}{dT}H_1^2 < 0.$

   Heat expelled from system: $\Delta Q = T_1 \Delta S.$

2. Adiabatic demagnetization: $\Delta S = 0 \;\Rightarrow\; S(T_1,H_1) = S(T_2,0).$

$$\Rightarrow \; S(T_2,0) = S(T_1,0) + \frac{1}{2}H_1^2\left.\frac{d\chi_T}{dT}\right|_{T_1} \;\Rightarrow\; T_2 < T_1.$$

Consider the entropy function $S(T,H)$ for iron ammonium alum.

- The sequence of steps 1 and 2 approaches absolute zero. As $T \to 0$, adiabates and isotherms become increasingly parallel, implying a diminishing efficiency of the cooling process.
- The sequence of steps requires heat reservoirs at various temperatures. They can be established by employing a Carnot engine consisting of steps 1 and 2 and their inverses.
- Magnetic refrigerators using paramagnetic salts attain ~ 0.2K. Adiabatic demagnetization attains mK temperatures.