Maxwell velocity distribution [tln38]

Criteria used by Maxwell:

- statistical independence: $f(v_x, v_y, v_z) = f_1(v_x)f_1(v_y)f_1(v_z)$.
- spherical symmetry: $f_1(v_x)f_1(v_y)f_1(v_z) = f_1\left(\sqrt{v_x^2+v_y^2+v_z^2}\right) f_1(0)f_1(0)$.
- equipartition: $\frac{1}{2}m\langle v_\alpha^2\rangle = \frac{1}{2}k_BT, \quad \alpha = x, y, z.$

Velocity distribution:

$$\Rightarrow\; f(v_x, v_y, v_z) = \left(\frac{m}{2\pi k_BT}\right)^{3/2} \exp\left(-\frac{m(v_x^2+v_y^2+v_z^2)}{2k_BT}\right).$$

Speed distribution:

integrate $f(v_x, v_y, v_z)$ over shell $v < \sqrt{v_x^2+v_y^2+v_z^2} < v + dv$.

$$\Rightarrow\; f_s(v) = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2k_BT}\right)^{3/2} v^2\, e^{-mv^2/2k_BT}.$$

Energy distribution:

Use $E = \frac{1}{2}mv^2, \quad v^2dv = \frac{1}{2}\left(\frac{2}{m}\right)^{3/2} E^{1/2}dE.$

$$\Rightarrow\; f_E(E) = \frac{2}{\sqrt{\pi}}(k_BT)^{-3/2}\sqrt{E}\, e^{-E/k_BT}.$$

Root-mean-square speed: $\sqrt{\langle v^2\rangle} = \sqrt{\frac{3k_BT}{m}}$.

Mean speed: $\langle v\rangle = \sqrt{\frac{8k_BT}{\pi m}}$.

Most frequent speed: $\left.\frac{df_s}{dv}\right|_{v_0} = 0 \;\Rightarrow\; v_0 = \sqrt{\frac{2k_BT}{m}}$.