Boltzmann equation [tln39]

How does an arbitrary nonequilibrium velocity distribution $f(\vec{v}, t)$ approach equilibrium? Boltzmann's kinetic equation takes into account elastic pair collisions, characterized by a scattering cross section $\sigma(\vec{v}_1, \vec{v}_2; \vec{v}_1', \vec{v}_2')$ that depends on the velocities of the two particles before and after the collision.

During the infinitesimal time interval τ, the number of particles with velocities $\vec{v}_1 d^3v_1$ changes due to contributions A and B from two kinds of processes:

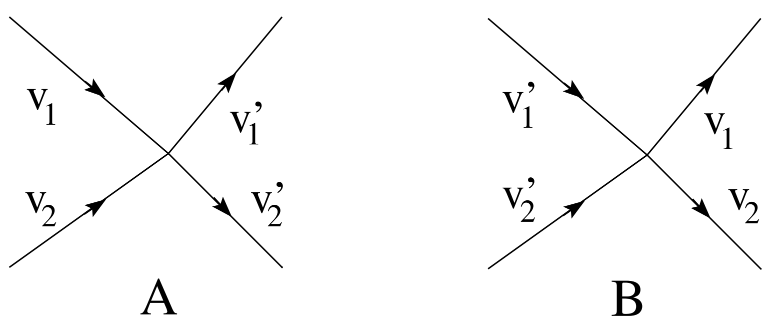
$$[f(\vec{v}_1, t+\tau) - f(\vec{v}_1, t)]\, d^3v_1 = B - A,$$

where the number of collisions away from $\vec{v}_1 d^3v_1$ is

$$A = \tau d^3v_1 \int d^3v_2 \int d^3v_1' \int d^3v_2'\, \sigma(\vec{v}_1, \vec{v}_2; \vec{v}_1', \vec{v}_2') f(\vec{v}_1, t) f(\vec{v}_2, t)$$

and the number of collisions into $\vec{v}_1 d^3v_1$ is

$$B = \tau d^3v_1 \int d^3v_2 \int d^3v_1' \int d^3v_2'\, \sigma(\vec{v}_1', \vec{v}_2'; \vec{v}_1, \vec{v}_2) f(\vec{v}_1', t) f(\vec{v}_2', t).$$



Here we have made the assumption of molecular chaos, which neglects correlations produced by the collisions: $f^{(2)}(\vec{v}_1, \vec{v}_2, t) = f(\vec{v}_1, t) f(\vec{v}_2, t)$.

Symmetry properties: $\sigma(\vec{v}_1, \vec{v}_2; \vec{v}_1', \vec{v}_2') = \sigma(\vec{v}_2, \vec{v}_1; \vec{v}_2', \vec{v}_1') = \sigma(\vec{v}_1', \vec{v}_2'; \vec{v}_1, \vec{v}_2)$.

Boltzmann equation for a spatially uniform velocity distribution:

$$\Rightarrow \frac{\partial}{\partial t} f(\vec{v}_1, t) = - \int d^3v_2 \int d^3v_1' \int d^3v_2'\, \sigma(\vec{v}_1, \vec{v}_2; \vec{v}_1', \vec{v}_2') \times [f(\vec{v}_1, t) f(\vec{v}_2, t) - f(\vec{v}_1', t) f(\vec{v}_2', t)].$$