H-theorem and irreversibility [tln41]

Q: How does the preferred time direction, selected by the monotonic time-dependence of $H(t)$, follow from the underlying microscopic dynamics, which is invariant under time reversal?

A: The solution $f(\vec{v}_1, t)$ of the Boltzmann equation is to be interpreted as representing the properties of an ensemble of systems, i.e. the average behavior of systems that are prepared equally (on a macroscopic level).

Consider the function $\tilde{H}(t) = \int d^3v_1\, \tilde{f}(\vec{v}_1, t) \ln \tilde{f}(\vec{v}_1, t)$,

calculated via computer simulation, where $\tilde{f}(\vec{v}_1, t)$ now represents the velocity distribution of a single system.

Simulation data show that $\tilde{H}(t)$ tends to decrease and approach an asymptotic value just as the function $H(t)$ does.

Effect of velocity inversion at time t_I: $\tilde{H}(t)$ increases at $t > t_I$ for some time, then decreases again and approaches the same asymptotic value as $H(t)$ does.

We can interpret $-\tilde{H}(t)$ as our uncertainty about the particle velocities in the system. The information contained in $\tilde{f}(\vec{v}_1, t)$ over and above the three general properties from which the Maxwell distibution was derived is $\tilde{H}(t) - \tilde{H}(\infty)$. However, this information is insufficient to carry out the velocity inversion.

Performing the velocity inversion requires an influx of information beyond what is contained in $\tilde{f}(\vec{v}_1, t)$, which causes a discontinuous drop in uncertainty of our knowledge about the particle velocities. At $t = t_I$, where the velocity inversion occurs, Boltzmann's function $H(t)$ jumps to a higher value and then decreases gradually as the information injected gets lost gradually in the wake of collisions.