Entropy of mixing revisited

Consider two dilute gases in a rigid and insulating box separated by a mobile conducting wall as in [tln25]: \( N_1 \) atoms on the left and \( N_2 \) atoms on the right. At thermal equilibrium: \( N_1/V_1 = N_2/V_2 \).

The removal of the internal wall initiates the mixing of particles 1 and 2.

Is the process reversible or irreversible? The answer depends on whether particles 1 and 2 are of the same kind (indistinguishable) or of a different kind (distinguishable).

Mixing occurs without changes in any of the following quantities:

- total internal energy: \( U = U_1 + U_2 \),
- total volume: \( V = V_1 + V_2 \),
- total number of particles: \( N = N_1 + N_2 \).

Consider the Sackur-Tetrode formula for the entropy of an ideal gas [tex73]:

\[
S(U, V, N) = \frac{5}{2} Nk_B + Nk_B \ln \left[ \frac{V}{Nh^3} \left( \frac{4\pi mU}{3N} \right)^{3/2} \right].
\]

**Distinguishable particles:** irreversible process

Initial entropy: \( S_{init} = S(U_1, V_1, N_1) + S(U_2, V_2, N_2) \)

Final entropy: \( S_{fin} = S(U_1 + U_2, V_1 + V_2, N_1 + N_2) \)

Entropy change: \( \Delta S = N_1k_B \ln \frac{V_1 + V_2}{V_1} + N_2k_B \ln \frac{V_1 + V_2}{V_2} > 0 \).

**Indistinguishable particles:** reversible process

Initial entropy: \( S_{init} = S(U_1, V_1, N_1) + S(U_2, V_2, N_2) \)

Final entropy: \( S_{fin} = S(U_1 + U_2, V_1 + V_2, N_1 + N_2) \)

Entropy change: \( \Delta S = 0 \).