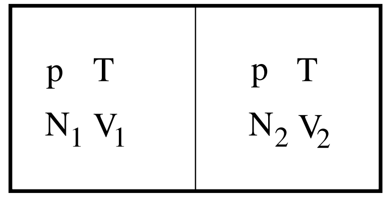
Entropy of mixing revisited [tln50]

Consider two dilute gases in a rigid and insulating box separated by a mobile conducting wall as in [tln25]: N_1 atoms on the left and N_2 atoms on the right.
At thermal equilibrium: $N_1/V_1 = N_2/V_2$.



The removal of the internal wall initiates the mixing of particles 1 and 2.

Is the process reversible or irreversible? The answer depends on whether particles 1 and 2 are of the same kind (indistinguishable) or of a different kind (distinguishable).

Mixing occurs without changes in any of the following quantities:

- total internal energy: $U = U_1 + U_2$,
- total volume: $V = V_1 + V_2$,
- total number of particles: $N = N_1 + N_2$.

Consider the Sackur-Tetrode formula for the entropy of an ideal gas [tex73]:

$$S(U, V, N) = \frac{5}{2} N k_B + N k_B \ln\left[\frac{V}{Nh^3}\left(\frac{4\pi m U}{3N}\right)^{3/2}\right].$$

Distinguishable particles: irreversible process

Initial entropy: $S_{init} = S(U_1, V_1, N_1) + S(U_2, V_2, N_2)$

Final entropy: $S_{fin} = S(U_1, V_1 + V_2, N_1) + S(U_2, V_1 + V_2, N_2)$

Entropy change: $\Delta S = N_1 k_B \ln\frac{V_1 + V_2}{V_1} + N_2 k_B \ln\frac{V_1 + V_2}{V_2} > 0.$

Indistinguishable particles: reversible process

Initial entropy: $S_{init} = S(U_1, V_1, N_1) + S(U_2, V_2, N_2)$

Final entropy: $S_{fin} = S(U_1 + U_2, V_1 + V_2, N_1 + N_2)$

Entropy change: $\Delta S = 0$.