Ideal Fermi-Dirac gas: heat capacity [tsl45]

Internal energy:

$$U = \frac{\mathcal{D}}{2} \mathcal{N} k_B T \frac{f_{\mathcal{D}/2+1}(z)}{f_{\mathcal{D}/2}(z)}.$$

Heat capacity [use $z g_n'(z) = g_{n-1}(z)$ for $n \geq 1$]:

$$\frac{C_V}{\mathcal{N} k_B} = \left(\frac{\mathcal{D}}{2} + \frac{\mathcal{D}^2}{4} \right) \frac{f_{\mathcal{D}/2+1}(z)}{f_{\mathcal{D}/2}(z)} - \frac{\mathcal{D}^2}{4} \frac{f'_{\mathcal{D}/2+1}(z)}{f'_{\mathcal{D}/2}(z)}.$$

Low-temperature asymptotic behavior:

$$\frac{C_V}{\mathcal{N} k_B} \sim \mathcal{D} \, \frac{\pi^2}{6} \, \frac{T}{T_F}.$$

High-temperature asymptotic behavior:

$$\frac{C_V}{\mathcal{N} k_B} \sim \frac{\mathcal{D}}{2} \left[1 - \frac{\mathcal{D}/2 - 1}{2^{\mathcal{D}/2-1} \Gamma(\mathcal{D}/2)} \left(\frac{T_F}{T} \right)^{\mathcal{D}/2} \right].$$