[tex10] Work extracted from finite heat reservoir in finite environment

A (finite) heat reservoir with heat capacity $C_H = \text{const}$ is initially at temperature T_H and the (finite) environment with heat capacity C_L at the lower temperature T_L.
(a) When heat is allowed to flow from the reservoir to the environment, both will end up at the temperature $T_f = (C_H T_H + C_L T_L)/(C_H + C_L)$ (arithmetic mean). Verify this and determine the total amount of heat ΔQ that has been transferred.
(b) When the reservoir is connected to the environment by a Carnot engine which absorbs an infinitesimal amount of heat δQ per cycle, converts part of it into work δW, and dumps the rest into the environment, the final common temperature of the reservoir and the environment will be $T_f = T_H^{C_H/(C_H+C_L)} T_L^{C_L/(C_H+C_L)}$ (geometric mean). Verify this and determine the total amout of work ΔW that has been extracted from the system. The fraction of the excess internal energy $U_{ex} = C_H(T_H - T_L)$ that can be converted into work is characterized by the quantity $\Delta W/U_{ex}$. Plot this quantity versus the reduced temperature $(T_H - T_L)/T_L$ for $C_H = C_L$ and $T_L < T_H < 3T_L$. Discuss the properties of this function in the limit $T_H \to T_L$.

Solution: