Consider the two-step cycle for a classical ideal gas \( pV = Nk_BT, C_V = \alpha Nk_B, \gamma \approx C_p/C_V = (\alpha + 1)/\alpha \) as shown. The first step (A) is an adiabatic compression and the second step (B) an expansion along a straight line segment in the \((V,p)\)-plane.

(a) Show that the difference in internal energy \( \Delta U = U_1 - U_2 \) is determined by the expression

\[
\Delta U = \frac{\Delta U}{p_1 V_1} = \alpha \left[ 1 - \left(\frac{V_1}{V_2}\right)^{\gamma-1} \right].
\]

(b) Show that the heat transfer \( \delta Q \) between system and environment during a volume increase from \( V \) to \( V + \delta V \) along the straight line segment is given by the expression

\[
\delta Q = \frac{\delta Q}{p_1 V_1} = \left[ (1 + \alpha)(1 + \sigma) - (1 + 2\alpha)\sigma \frac{V}{V_1} \right] dV, \quad \sigma = \frac{1 - (V_1/V_2)^\gamma}{V_2/V_1 - 1}.
\]

(c) Show that along the straight-line segment the system absorbs heat if \( V_1 < V < V_c \) and expels heat if \( V_c < V < V_2 \), where \( V_c/V_1 = [(1 + \alpha)(1 + \sigma)]/[(1 + 2\alpha)\sigma] \).

Solution: