

[tex110] Occupation number fluctuations

Consider an ideal quantum gas specified by the grand partition function Z . Start from the expressions

$$\langle n_k^2 \rangle - \langle n_k \rangle^2 = \frac{1}{Z} \beta^{-2} \frac{\partial^2 Z}{\partial \epsilon_k^2} - \left[\frac{1}{Z} \beta^{-1} \frac{\partial Z}{\partial \epsilon_k} \right]^2, \quad \ln Z = \frac{1}{a} \sum_{k=1}^{\infty} \ln(1 + a z e^{-\beta \epsilon_k}),$$

where $a = +1, 0, -1$ represent the FD, MB, and BE cases, respectively, to derive the following result for the relative fluctuations in the occupation numbers:

$$\frac{\langle n_k^2 \rangle - \langle n_k \rangle^2}{\langle n_k \rangle^2} = \frac{1}{\langle n_k \rangle} - a.$$

Note that in the BE (FD) statistics, these fluctuations are enhanced (suppressed) relative to those in the MB statistics.

Solution: