[tex112] Maxwell-Boltzmann gas in $\mathcal{D}$ dimensions

From the expressions for the grand potential and the density of energy levels of an ideal Maxwell-Boltzmann gas in $\mathcal{D}$ dimensions and confined to a box of volume $V = L^{\mathcal{D}}$ with rigid walls,

$$\Omega(T, V, \mu) = -k_B T \sum_k e^{-\beta(\epsilon_k - \mu)}, \qquad D(\epsilon) = \frac{L^{\mathcal{D}}}{\Gamma(\mathcal{D}/2)} \left(\frac{m}{2\pi\hbar^2}\right)^{\mathcal{D}/2} \epsilon^{\mathcal{D}/2-1},$$

derive the familiar results $pV = \mathcal{N} k_B T$ for the equation of state, $C_{V\mathcal{N}} = (\mathcal{D}/2)\mathcal{N} k_B$ for the heat capacity, and $pV^{(\mathcal{D}+2)/\mathcal{D}} = \text{const}$ for the adiabate at fixed $\mathcal{N}$.

Solution: