(a) From the fundamental thermodynamic relations for the Bose-Einstein gas in $D$ dimensions (see [tln67]), derive the following parametric expression for the isochore at $T \geq T_c$:

$$
\frac{p}{p_v} = \frac{g_{D/2+1}(z)}{[g_{D/2}(z)]^{2/D+1}}, \quad \frac{T}{T_v} = \left[ g_{D/2}(z) \right]^{-2/D},
$$

where $k_B T_v = \Lambda v^{-2/D}$ and $p_v = \Lambda v^{-2/D+1}$ with $\Lambda = h^2/2\pi m$ are convenient reference values.

(b) Calculate the leading correction to the Maxwell-Boltzmann result at high temperature. 

(c) Calculate the exact dependence of $p/p_v$ on $T/T_v$ at $T \leq T_c$ in $D > 2$. Show that this result also holds asymptotically for $T \ll T_v$ in dimensions $D = 1$ and $D = 2$.

**Solution:**