From the fundamental thermodynamic relations for the Bose-Einstein gas in \( D > 2 \) dimensions (see [th67]), derive the following expressions for the isotherm at \( v > v_c \) and the isobar at \( T \leq T_c \):

\[
\frac{p}{p_T} = g_{D/2+1}(z), \quad \frac{v}{v_T} = [g_{D/2}(z)]^{-1};
\]

\[
\frac{v}{v_p} = \left[ g_{D/2+1}(z) \right]^{D/(D+2)} g_{D/2}(z), \quad \frac{T}{T_p} = [g_{D/2+1}(z)]^{-2/(D+2)}.
\]

where \( v_T = (\Lambda/k_B T)^{D/2} \), \( p_T = \Lambda(k_B T/\Lambda)^{D/2+1} \), \( k_B T_p = \Lambda(p/\Lambda)^{2/(D+2)} \), \( v_p = (\Lambda/p)^{D/(D+2)} \) with \( \Lambda \equiv h^2/2\pi m \) are convenient reference values for temperature and pressure and reduced volume. (b) Calculate the leading correction to the Maxwell-Boltzmann result for the isotherm at low density and for the isobar at high temperature.

Solution: