Consider the isochore of an ideal Fermi-Dirac gas in $D$ dimensions, as given by the parametric relation

$$\frac{p}{p_v} = \frac{T}{T_v} \frac{f_{D/2+1}(z)}{f_{D/2}(z)}, \quad \frac{T}{T_v} = \left[ f_{D/2}(z) \right]^{-2/D}. $$

where $k_B T_v = \Lambda / v^{2/D}$, $p_v = k_B T / v$, $\Lambda \equiv h^2 / 2\pi m$, $v \equiv gV / N$. The upward deviation of this result from the Maxwell-Boltzmann result, $p / p_v = T / T_v$, is a manifestation of repulsive statistical interaction between fermions. (a) Calculate the high-$T$ asymptotic dependence of $p / p_v$ on $T / T_v$ including the leading correction to MB behavior. (b) Calculate the low-$T$ limit of $p / p_v$. (c) Calculate the low-$T$ limit of $p / p_F$, where $T_F = T_v \left[ \Gamma(D/2 + 1) \right]^{2/D}$ is the Fermi temperature and $p_F = k_B T_F / v$ the associated reference pressure. (d) Compare the differently scaled statistical interaction pressures $p / p_v$ and $p / p_F$ at $T = 0$ in the limit $D \rightarrow \infty$.

Solution: