[tex119] FD gas in $\mathcal{D}$ dimensions: statistical interaction pressure

Consider the isochore of an ideal Fermi-Dirac gas in $\mathcal{D}$ dimensions, as given by the parametric relation

$$\frac{p}{p_v} = \frac{T}{T_v}\frac{f_{\mathcal{D}/2+1}(z)}{f_{\mathcal{D}/2}(z)}, \qquad \frac{T}{T_v} = \left[f_{\mathcal{D}/2}(z)\right]^{-2/\mathcal{D}}.$$

where $k_B T_v = \Lambda/v^{2/\mathcal{D}}$, $p_v = k_B T/v$, $\Lambda \doteq h^2/2\pi m$, $v \doteq gV/\mathcal{N}$. The upward deviation of this result from the Maxwell-Boltzmann result, $p/p_v = T/T_v$, is a manifestation of repulsive statistical interaction between fermions. (a) Calculate the high-T asymptotic dependence of p/p_v on T/T_v including the leading correction to MB behavior. (b) Calculate the low-T limit of p/p_v. (c) Calculate the low-T limit of p/p_F, where $T_F = T_v[\Gamma(\mathcal{D}/2+1)]^{2/\mathcal{D}}$ is the Fermi temperature and $p_F = k_B T_F/v$ the associated reference pressure. (d) Compare the differently scaled statistical interaction pressures p/p_v and p/p_F at $T = 0$ in the limit $\mathcal{D} \to \infty$.

Solution: