

[tex126] Quantum harmonic oscillators (microcanonical ensemble II)

Consider an array of  $N$  quantum harmonic oscillators:

$$H = \sum_{j=1}^N \left[ \hbar\omega \left( n_j + \frac{1}{2} \right) \right], \quad n_j = 0, 1, 2, \dots$$

(a) Calculate the entropy  $S(U, N)$  of this system in the *microcanonical* ensemble via saddle point method as follows. Express the number of distinct microstate at energy  $U$  in the form  $N_{\Delta} = \sum_{n_1} \cdots \sum_{n_N} \delta(U - H)$  with the  $\delta$ -function replaced by its Fourier integral. Then use the asymptotic Laplace expression for the integral and evaluate it retaining only contributions that are significant in the thermodynamic limit.

(b) Derive the internal energy  $U(T, N)$ , and the heat capacity  $C = (\partial U / \partial T)_N$  from  $S(U, N)$ .

**Solution:**