

[tex13] Room heater: Electric radiator versus heat pump

A room is to be kept at temperature $T_H = 294\text{K}$, (21°C). The outdoor temperature is T_L . Heat, which leaks through the windows and doors at the rate $\bar{Q}_{leak} = \gamma\Delta T$, must be replaced by a room heater at the same rate. The electric radiator converts electric power \bar{W}_{el} into heat \bar{Q}_{el} with 100% efficiency. The electric heat pump uses the amount \bar{W}_{sup} of electric power to drive a Carnot cycle in the reverse, which extracts heat \bar{Q}_L at temperature T_L from the exterior and converts it (reversibly) into heat $\bar{Q}_H = \bar{Q}_L + \bar{W}_{hp}$ at temperature T_H . In the relation $\bar{W}_{hp} = (1 - \lambda)\bar{W}_{sup}$, λ represents the energy loss in the gears of the heatpump. Quantities with overbars denote energy transfers per time unit.

- Find \bar{W}_{el} as a function of γ, T_H, T_L , and \bar{W}_{sup} as a function of $\gamma, \lambda, T_H, T_L$.
- Plot \bar{W}_{el}/γ and \bar{W}_{sup}/γ versus $t_L \equiv T_L - 273\text{K}$ (measured in $^\circ\text{C}$) for fixed $T_H = 294\text{K}$ and $\lambda = 0.8$ (20% efficiency).
- Determine the range of T_L where the heat pump is more economical than the radiator.

Solution: