Circular heat engine II

Consider 1 mol of a monatomic classical ideal gas \( pV = RT, \ U = \frac{3}{2}RT \) confined to a cylinder by a piston. The cylinder is in thermal contact with a heat bath of adjustable temperature. As the piston moves back and forth between volume \( V = V_0(1 - r) \) and \( V = V_0(1 + r) \) quasistatically, the temperature of the gas is being adjusted via thermal contact such that the cycle becomes circular in the \((V,p)\)-plane and proceeds in clockwise direction (\( \phi \) from 0 to \( 2\pi \)).

(a) Calculate the rate \( \frac{dW}{d\phi} \) at which work is being performed, the rate \( \frac{dU}{d\phi} \) at which the internal energy changes, and the rate \( \frac{dQ}{d\phi} \) at which heat is being transferred.

(b) Set \( r = 0.5 \) and identify the segments along the circle where each rate is positive or negative.

(c) Repeat the previous part for \( r = 0.9 \).

(d) Plot all three rates as functions of \( \phi/\pi \) for \( r = 0.5 \) in one graph and then for \( r = 0.9 \) in a second graph.

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\begin{array}{c}
p/p_0 \\
\hline
1 & \phi \\
\hline
0 & V/V_0
\end{array}
\]

Solution: