

[tex150] Isothermal atmosphere

Consider a column of air [molar mass $M = 29\text{g}$] treated as a classical ideal gas [$pV = nRT$] in a uniform gravitational field $g = 9.81\text{m/s}^2$. The column is assumed to be in thermal equilibrium.

(a) Calculate the dependence of pressure p on height z and (uniform) temperature T , assuming that the pressure is p_0 at $z = 0$.

(b) At what height z_1 (in meters) has the pressure fallen to half of p_0 and at what height z_2 to one percent of p_0 if the temperature is 20°C everywhere?

Hint: Start from the relation, $dp(z) = -\rho(z)d\mathcal{U}(z)$, between pressure p , mass density ρ , and gravitational potential \mathcal{U} at height z . This relation expresses the increment of pressure caused by the weight of a thin layer of air. The ideal-gas equation of state is assumed to hold locally at all heights.

Solution: