[tex154] Ultrarelativistic classical ideal gas in two dimensions

Consider a classical ideal gas of N particles confined to a two-dimensional box of area V in thermal equilibrium at extremely high temperature T. Most particles are moving at speeds close to the speed of light c. We describe this system by a Hamiltonian of the form,

$$H = \sum_{l=1}^{N} \sqrt{p_x^2 + p_y^2}\, c.$$

(a) Show that the canonical partition function is

$$Z_N = \frac{1}{N!}\left[2\pi V \left(\frac{k_B T}{hc}\right)^2\right]^N.$$

(b) Find the Helmholtz free energy $A(T,V,N)$, the entropy $S(T,V,N)$, the pressure $p(T,V,N)$, and the internal energy $U(T,N)$.
(c) Find the adiabate (for constant N) and express it in the form $p^\nu V = \text{const}$.
(d) Infer from the given canonical partition function $Z_N(T,V)$ an explicit expression for the grand partition function $\mathcal{Z}(T,V,\mu)$, where $\mu = k_B T \ln z$ is the chemical potential and z is the fugacity.
Use $\int_0^\infty dx x^n e^{-ax} = n! a^{-n-1}$, $\quad \ln n! \simeq n\ln n - n$, $\quad \sum_{n=0}^{\infty} x^n/n! = e^x$.

Solution: