Thermodynamics of an ideal paramagnet III

For an ideal Langevin paramagnet, which is specified by the equation of state $M = \tanh(H/T)$ (Langevin function) and the internal energy $U \equiv 0$, find (a) the entropy $S(T, H)$ and the enthalpy $E(T, H)$; (b) the thermodynamic potentials $A(T, M)$, $G(T, H)$; (c) the response functions $\chi_T$, $\chi_S$, $\alpha_H$, $C_H$. Determine the integration constant $S_0$ in $S(T, H)$ such that $S \to 0$ for $T \to 0$ and $H \neq 0$ in accordance with the third law of thermodynamics.

Solution: